Measurement of Periodic Dispersion

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January 26, 2007

At fixed field, measure the frequency and take equilibrium orbits for various settings of the radius parameter in the RF Beam Control program. Let R and f be the radius and frequency measured at the nominal setting of the radius parameter. We call these the nominal radius and frequency. The nominal velocity is then

$$v = 2\pi R f \tag{1}$$

which gives

$$\beta = v/c, \quad \gamma = 1/\sqrt{1-\beta^2} \tag{2}$$

and nominal momentum

$$p = mc\beta\gamma \tag{3}$$

where m is the particle mass and c is the velocity of light. For these nominal values we take an equilibrium orbit and call it the nominal orbit.

The departures dR, df, dp, and dB of the radius, frequency, momentum, and field from their nominal values are given by the differential relations

$$\frac{dp}{p} = \gamma^2 \left\{ \frac{df}{f} + \frac{dR}{R} \right\}, \quad \frac{dR}{R} = \frac{1}{\gamma_t^2} \left\{ \frac{dp}{p} - \frac{dB}{B} \right\} \tag{4}$$

where B is the nominal field and γ_t is transition gamma. At fixed field we have $dB/B = 0$ and

$$\frac{dR}{R} = \frac{1}{\gamma_t^2} \left\{ \frac{dp}{p} \right\}. \tag{5}$$

Thus

$$\frac{dp}{p} = \gamma^2 \left\{ \frac{df}{f} + \frac{dR}{R} \right\} = \gamma^2 \left\{ \frac{df}{f} + \frac{1}{\gamma_t^2} \frac{dp}{p} \right\} \tag{6}$$

$$\left\{ \frac{1}{\gamma^2} - \frac{1}{\gamma_t^2} \right\} \frac{dp}{p} = \frac{df}{f}, \quad \left\{ \gamma_t^2 - \gamma^2 \right\} \frac{dp}{p} = \left\{ \gamma_t^2 \gamma^2 \right\} \frac{df}{f} \tag{7}$$

and

$$\frac{dp}{p} = \left\{ \frac{\gamma_t^2 \gamma^2}{\gamma_t^2 - \gamma^2} \right\} \frac{df}{f}. \tag{8}$$

From the measured frequencies we therefore obtain dp/p. For each value of dp/p we take a "difference orbit" which gives the departure of the equilibrium orbit from the nominal one (obtained with $dp/p = 0$). The difference orbit divided by dp/p then gives the periodic dispersion.

Note that the design radii R_B, R_A, R_R in Booster, AGS, and RHIC are (in meters)

$$R_B = 201.780/(2\pi), \quad R_A = 128.4526, \quad R_R = 3833.845181/(2\pi). \tag{9}$$

The design values for γ_t in Booster, AGS, and RHIC are 4.885, 8.5, and 22.89 respectively.